

## Section 11.7 - Strategy for Testing Series

1. If it is easy to see that  $\lim_{n \rightarrow \infty} a_n \neq 0$ , use the  $n^{\text{th}}$  term test for divergence. Remember that  $\lim_{n \rightarrow \infty} a_n = 0$  does not imply convergence.
2. If the series is of the form  $\sum \frac{1}{n^p}$ , it is a  $p$ -series (which converge for  $p > 1$  and diverge for  $p \leq 1$ ).
3. If the series is of the form  $\sum ar^n$ , it is geometric, and thus converges if  $|r| < 1$ , and diverges if  $|r| > 1$ . Its sum is  $\frac{\text{first term}}{1-r}$ .
4. If the series has an alternating factor  $(-1)^n$ , it is an alternating series of the form  $\sum_{n=0}^{\infty} (-1)^n b_n$ , and converges if for all  $n \geq 0$ ,  
(i)  $b_n \geq 0$ , (ii)  $b_{n+1} \leq b_n$ , (iii)  $\lim_{n \rightarrow \infty} b_n = 0$
5. If the series has a form similar to  $p$ -series or a geometric series, a comparison test should be used. Both comparison tests below require positive terms.

### • Direct Comparison Test (DCT)

(a) if  $a_n \leq b_n$  for all  $n$ , and  $\sum b_n$  converges, then  $\sum a_n$  converges.

(b) if  $a_n \geq b_n$  for all  $n$ , and  $\sum b_n$  diverges, then  $\sum a_n$  diverges.

### • Limit Comparison Test (LCT)

If  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = C$ , with  $0 < C < \infty$ , then either both series ( $\sum a_n$  and  $\sum b_n$ ) converge, or they both diverge.

Remark: if  $\sum a_n$  has some negative terms, we can test for absolute convergence by looking at  $\sum_{n=1}^{\infty} |a_n|$ : Abs. Conv  $\Rightarrow$  Convergence

6. Series that involve factorials and/or exponentials are conveniently tested using the Ratio test: Let  $L = \lim \left| \frac{a_{n+1}}{a_n} \right|$ ; Then

(i) if  $L < 1$ ,  $\sum a_n$  converges absolutely, and thus converges.

(ii) if  $L > 1$ ,  $\sum a_n$  diverges. (iii) if  $L = 1$ , test is inconclusive!

Remark: P-series or Series with rational functions as  $a_n$  always yield a ratio of 1.

7. If  $a_n = f(n)$ , where  $\int_1^{\infty} f(x) dx$  is easily evaluated, the integral test may be used: if  $f(x)$  is positive, continuous, and decreasing on  $[1, \infty)$ :

(i)  $\sum a_n$  converges if  $\int_1^{\infty} f(x) dx$  converges,

(ii)  $\sum a_n$  diverges if  $\int_1^{\infty} f(x) dx$  diverges.